Journal of Sound and Vibration (2000) **233**(3), 441–453
doi:10.1006/jsvi.1999.2820, available online at http://www.idealibrary.com on IDEAL®

THEORETICAL COMPARISON OF MOTIONAL AND TRANSFORMER EMF DEVICE DAMPING EFFICIENCY

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(*Received* 21 *December* 1998, *and in final form* 4 *November* 1999)

In this paper, theoretical comparison between electromagnetic dampers based on a "motional emf" and "transformer emf" design is presented. Transformer emf devices are based on the generation of emf in a stationary circuit, in which the emf is generated by a time-varying magnetic field linking the circuit. Motional emf devices are based on the generation of emf due to a moving conductor within a stationary magnetic field. Both of these designs can be used as damping elements for applications such as semi-active and regenerative vehicle suspension systems. The findings herein are provided so as to evaluate the most efficient device for such applications. The analysis consists of comparing the damping coefficient of the electromagnetic devices for a given magnetic field and given volume of conducting material. It has been found that for a limited range of dimensions, the transformer emf devices can be more then 1·2 times as efficient as the motional emf devices. However, for most realistic situations, motional emf devices will have the highest efficiency.
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1. INTRODUCTION

Previous research has analysed the use of electromagnetic devices as dampers within vehicle suspension systems. Specifically, these devices have been proposed for use as regenerative dampers [1, 2]; semi-active dampers [3]; semi-active dampers with energy regeneration [4–7], and for active vibration control [8]. All of these electromagnetic dampers provide a damping force due to the movement of a closed conducting circuit through a stationary magnetic field. The damping force is provided by a generated current, which is developed from the induced potential within a conducting circuit. The potential, or generated "emf" arising from this method is defined as "motional emf" [9]. Therefore, these devices are defined as "motional emf" electromagnetic devices. Examples of these devices include DC machines, in which a conducting coil rotates within a stationary magnetic field.

This is not the only method of producing an emf within a closed conducting circuit. An emf can also be generated using "transformer emf". Transformer emf electromagnetic devices generate an emf within a stationary conducting circuit, due to a time-varying magnetic field linking the circuit [9]. Examples of these devices include bicycle "dynamo" generators, in which the movement of alternating-polarity magnetic poles causes a time-varying magnetic field in the core of a conducting coil of wire.

The research, documented herein, demonstrates how transformer emf devices can be used as damping elements. It also presents a comparison between the damping efficiency of transformer emf and motional emf electromagnetic devices. The damping efficiency is defined as the maximum damping coefficient for a given magnetic field and volume of

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conducting material. This analysis is presented in order to determine which device is more appropriate for use in damping applications.

2. MOTIONAL EMF DEVICES

Motional emf electromagnetic devices are based on the principle of a damping force produced when a given volume of conducting material moves through a stationary magnetic field. Karnopp [3] analyzed the motional emf, moving-coil damper which can provide a variable damping coefficient for use in road vehicle suspension systems. The analysis, presented here, is generalized for any given volume of conducting material moving within a magnetic field, independent of the device topology. Figure 1 shows the general volume of conducting material V, moving with velocity $\dot{X}$, within a stationary magnetic field, B_0. The current flowing in each conducting element is defined as i.

The electrical equivalent circuit for the device in Figure 1 is shown in Figure 2. There are N_C identical conductors of length h. The resistance r is the resistance of each separate conducting length, and R_{ext} is the external resistance. The emf, E_C is produced in each element for a given velocity.

The voltage around any loop, which includes the external resistance R_{ext}, is given in

$$E_C = ir + IR_{ext} \quad \text{(V)}, \tag{1}$$

which is equivalent to

$$E_C = I(r/N_C + R_{ext}) \quad \text{(V)}, \tag{2}$$

where the internal device resistance is given by

$$r/N_C = R_{int} \quad (\Omega). \tag{3}$$

The total force, in Newtons (N), produced by the device F, and the emf generated in each conductor E_C is given in equations (4) and (5) respectively [9],

$$F = N_C B_0 hi \quad \text{(N)}, \tag{4}$$

$$E_C = B_0 h\dot{X} \quad \text{(V)}. \tag{5}$$

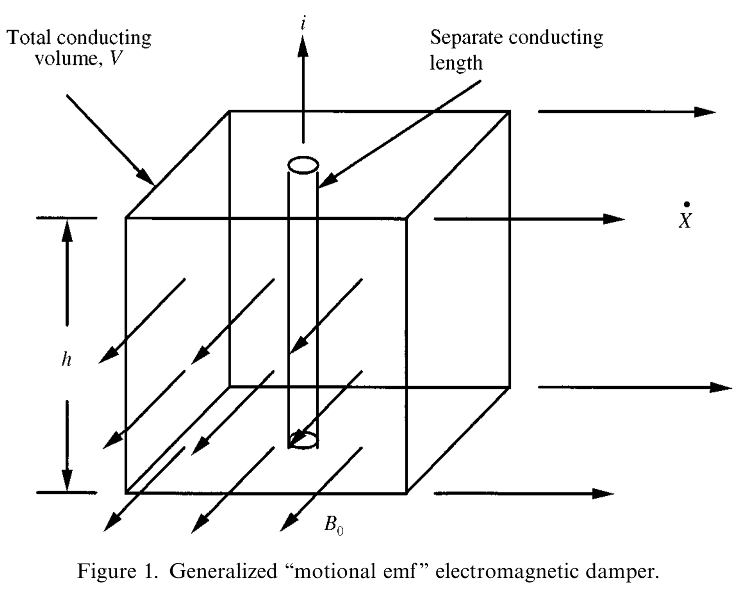

Figure 1. Generalized “motional emf” electromagnetic damper.

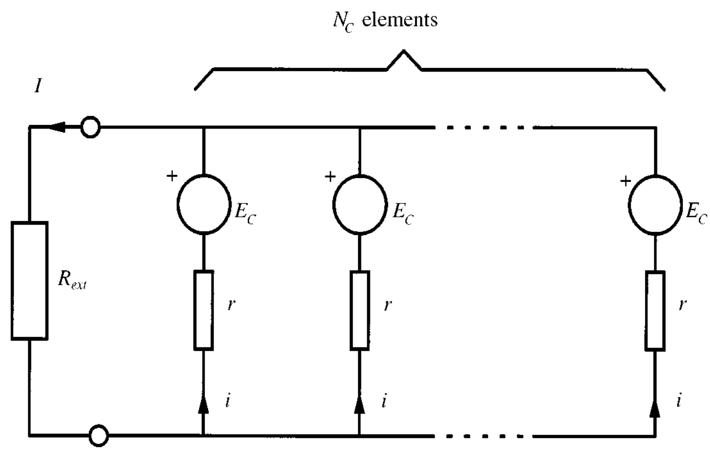

Figure 2. Electrical equivalent "motional emf" damper.

The damping coefficient of the motional emf device $Cmot$ is, therefore,

$$Cmot = \frac{F}{\dot{X}} = B_0^2 h^2 \frac{1}{R_{int} + R_{ext}} \quad (\text{N s/m}). \tag{6}$$

The maximum damping coefficient $Cmot_{max}$ occurs for an external resistance R_{ext}, of zero. The damping coefficient can now be represented in equation (7), where V is the volume of the conducting material, and σ is the conductivity of the conducting material,

$$Cmot_{max} = \frac{B_0^2 V}{\sigma} \quad (\text{N s/m}). \tag{7}$$

The damping coefficient has been given with respect to the total volume of the conducting material and the magnetic field. It is now possible to compare the damping coefficient with respect to these parameters for the transformer emf device in order to compare the damping efficiencies.

3. CIRCULAR TRANSFORMER EMF DEVICES

Equation (8) shows Faraday's law, which gives the emf V_{tran}, induced in a stationary closed circuit of N turns, with respect to the magnetic flux Φ, linking the circuit [9],

$$V_{tran} = -N \frac{\mathrm{d}\Phi}{\mathrm{d}t} \quad (\text{V}). \tag{8}$$

Figure 3 shows a generalized topology of this situation in which the N turns of conducting material are wound around a circular ferromagnetic core which contains a time-varying magnetic field. The core has a cross-sectional area A, and radius r_C. The magnetic field $B(t)$ is directed along the core in the direction of the vector $\mathbf{a}_S$. The width of the conducting coil is defined as w, and the length of the devices is l_C.

It is assumed that the magnetic field is uniform within the core, and has a magnitude varying sinusoidally with a frequency f_B, and maximum amplitude B_0, as

$$B(t) = B_0 \sin(2\pi f_B t)\, \mathbf{a}_S \quad (\text{T}). \tag{9}$$

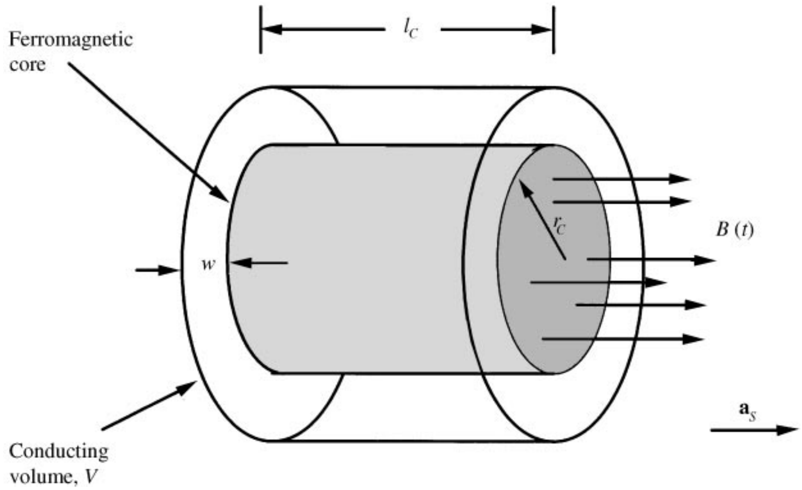

Figure 3. Coil of transformer emf damper.

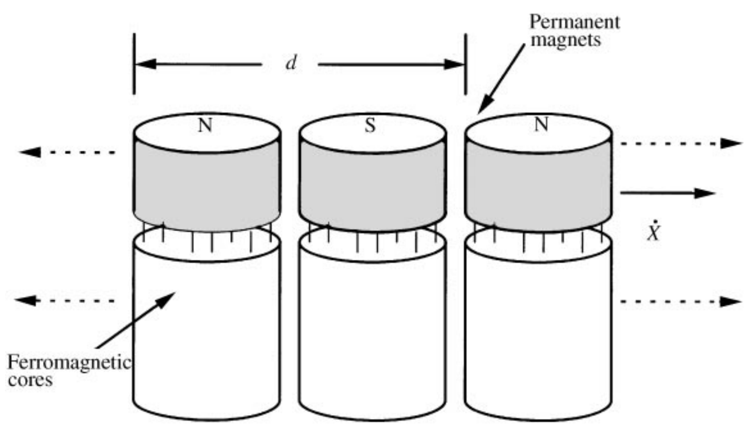

Figure 4. Core and permanent magnets in generalized circular "transformer emf" device.

This condition can be viewed with respect to a bicycle dynamo generator. In this device, the magnetic field within one or more cores changes due to the movement of magnetic devices with respect to the core. The magnetic devices provide a magnetic field that has a magnitude that varies as a function of position. An example of this is shown in Figure 4, in which a number of permanent magnets with alternating north–south pole faces moves, with velocity $\dot{X}$, relative to the ferromagnetic cores.

Figure 4 establishes the assumption that the amplitude of the time-varying magnetic field has an amplitude of B_0. This is because the maximum magnetic field is only supplied to the core when the magnets and cores line up. It is assumed that this maximum magnetic field is the same as the magnetic field supplied for the "motional emf" devices. Using these assumptions, the magnetic flux density, as a function of time t, within each core is

$$\Phi(t) = AB_0 \sin(2\pi f_B t) \quad \text{(Wb)}. \tag{10}$$

Therefore, from the substitution of equation (10) into equation (8), the induced emf V_{tran} in the conducting circuit is

$$V_{tran} = -NAB_0\, 2\pi f_B \cos(2\pi f_B t) \quad \text{(V)}. \tag{11}$$

From the induced emf, equation (12) shows the total power dissipated in the device P_{tran}, with an internal coil resistance R_{int}, attached to an external resistance R_{ext},

$$P_{tran} = \frac{[NAB_0 2\pi f_B \cos(2\pi f_B t)]^2}{(R_{int} + R_{ext})} \quad \text{(W)}. \tag{12}$$

The average power dissipation $\bar{P}_{tran}$ is given in equation (13), where $T(= 1/f_B)$ is the period of the time-varying magnetic field.

$$\bar{P}_{tran} = \frac{1}{T}\int_{t=\tau}^{\tau+T} P_{tran}\,\mathrm{d}t \quad \text{(W)}. \tag{13}$$

Also, the time-averaged cosine-squared function is given as

$$\frac{1}{T}\int_{t=t1}^{t1+T} \cos^2(2\pi f_B t)\,\mathrm{d}t = 0{\cdot}5. \tag{14}$$

Substituting equations (13) and (14) into equation (12) leads to the average power dissipation

$$\bar{P}_{tran} = \frac{[NAB_0]^2\, 2\pi^2}{(R_{int} + R_{ext})} f_B^2 \quad \text{(W)}. \tag{15}$$

From Figure 3, it can be shown that for the coils moving at velocity $\dot{X}$ relative to the permanent magnets, where d represents the distance between two "same-polarity" permanent magnets, the frequency of the magnetic field through the coils is given by

$$f_B = \frac{\dot{X}}{d} \quad \text{(1/s)}. \tag{16}$$

It is now possible to evaluate the power dissipation with respect to the relative device velocity. This is given by

$$\bar{P}_{tran} = \frac{2\pi^2}{d^2}\frac{[NAB_0]^2}{(R_{int} + R_{ext})}\dot{X}^2 \quad \text{(W)}. \tag{17}$$

Given that in a dissipative, physical system, the power P, dissipated is related to the force $\mathbf{f}$, and velocity $\dot{\mathbf{x}}$, vectors by,

$$P = \mathbf{f}\cdot\dot{\mathbf{x}} \quad \text{(W)}. \tag{18}$$

Therefore, the force produced by the transformer emf device is proportional to the relative velocity across the device, as shown in equation (19), and as the power is disssipated, the force is directed in the opposite direction to the velocity.

$$F = \frac{2\pi^2}{d^2}\frac{[NAB_0]^2}{(R_{int} + R_{ext})}\dot{X} \quad \text{(N)}. \tag{19}$$

Equation (19) reveals that the transformer emf device works as a "viscous" damper, and can therefore be used as a variable damper for applications such as vehicle suspension systems. It is possible to evaluate the equivalent, average damping coefficient $\bar{C}$, of the transformer emf device from

$$\bar{P} = \bar{C}\dot{X}^2 \quad \text{(W)}. \tag{20}$$

Using equations (17)–(20), the equivalent damping coefficient of the transformer emf device *Ctran* is defined as the time-averaged damping coefficient, and is

$$Ctran = \frac{2\pi^2}{d^2}\frac{[NAB_0]^2}{(R_{int} + R_{ext})} \quad \text{(N s/m)}. \tag{21}$$

The maximum damping coefficient $Ctran_{max}$ occurs for an external resistance R_{ext}, of zero, and is given by

$$Ctran_{max} = \frac{2\pi^2}{d^2}\frac{[NAB_0]^2}{R_{int}} \quad \text{(N s/m)}. \tag{22}$$

From Figure 5, the distance d is minimized according to

$$d = 4(r_C + w) \quad \text{(m)}. \tag{23}$$

The assumption is made that the conducting coil completely fills the volume defined as the coil region. Therefore, the conductor area a, the number of conducting turns N, the core length l_C, and the coil width w, can be related by

$$Na = l_C w \quad (\text{m}^2). \tag{24}$$

Also the surface area of the core A is given by

$$A = \pi r_C^2 \quad (\text{m}^2). \tag{25}$$

The total conductor length l is given by

$$l = N2\pi(r_C + w/2) \quad \text{(m)}, \tag{26}$$

and the internal conductor resistance R_{int} is given by

$$R_{int} = \frac{l\sigma}{a} \quad (\Omega), \tag{27}$$

where σ is the conductivity of the conducting material. Substituting equations (23)–(27) into the damping coefficient function, equation (20) takes the form

$$Ctran = \frac{\pi^3 l_C w r_C^4}{16(r_C + w)^2(r_C + w/2)}\frac{B_0^2}{\sigma} \quad \text{(N s/m)}. \tag{28}$$

Given that the actual volume of conducting material for the transformer emf device is given by

$$V_{ACT} = 2\pi l_C w(r_C + w/2) \quad (\text{m}^3), \tag{29}$$

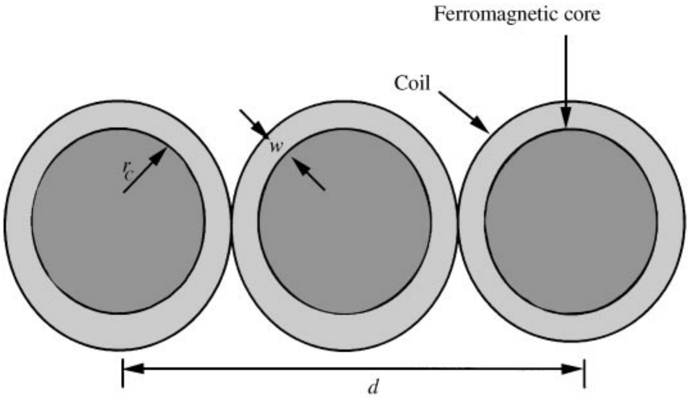

Figure 5. "Top view" of transformer emf device.

the efficiency of the transformer emf device, compared to the motional emf device, can be obtained by taking the ratio of their respective damping coefficients, for the condition that the motional emf device has a device volume is given in equation (29). This ratio is defined as H_C, and is given by

$$H_C = \frac{Ctran}{Cmot} = \frac{r_C^4 \pi^2}{32(r_C + w)^2 (r_C + w/2)^2}. \tag{30}$$

In order to simplify the analysis, a parameter, r_w, is defined as the normalized conducting coil width, or the ratio of the width of the conducting coil w, and the radius of the coil r_C, and is given by

$$r_w = w/r_C. \tag{31}$$

Substituting the ratio given in equation (31) into equation (30) gives the relative efficiency between the two devices with respect to the normalized coil width,

$$H_C = \frac{\pi^2}{32(1 + r_w)^2 (1 + r_w/2)^2}. \tag{32}$$

The relationship between the relative device efficiency H_C, and the normalized coil thickness r_w, is shown in Figure 6.

Figure 6 reveals that a maximum relative efficiency between the circular transformer emf device and the motional emf device occurs when the normalized coil thickness is zero, or when the width of the conducting coils is zero. The maximum efficiency for the circular transformer emf device is $\pi^2/32$, or 30·84%.

One disadvantage of the transformer emf device is due to the averaging of the time-varying magnetic field within the core. Equation (14) revealed that if the magnetic field varies sinusoidally, the average power dissipation, and, therefore, the damping coefficient, reduces by half, in comparison to a device such as the motional emf device, which continually uses the maximum magnetic field to produce a damping force.

The reduced relative device efficiency H_C for the circular transformer emf design with respect to the relative width of the conducting coil occurs due to the efficiency at which the device uses the available magnetic field. As the relative conducting coil width r_w increases for a given conductor volume and magnetic field, the change in magnetic field within the

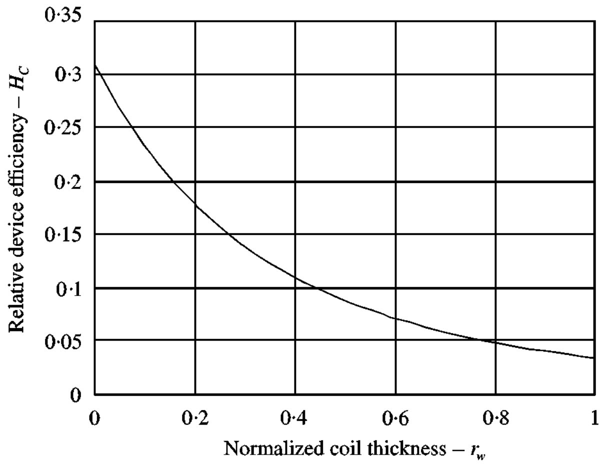

Figure 6. Circular "transformer emf" device efficiency.

core, for a given device velocity, reduces. Equation (15) reveals that this leads to a reduction in power dissipation, and therefore damping coefficient, for a given conductor volume and magnetic field.

From this analysis, it can be seen that the efficiency of the circular transformer emf device, relative to the motional emf device, is reasonably low. It is possible, however, to modify the dimensions of the transformer emf design, in order to improve the relative efficiency of the device. This is discussed in the following section.

4. RECTANGULAR TRANSFORMER EMF DEVICES

Figure 7 shows the design of a rectangular transformer emf device. This design is based on the same principle as the design shown in Figure 3, in which the N turns of conducting material are wound around the ferromagnetic core which contains a time-varying magnetic field. The core has a cross-sectional area A, and side lengths of l_1 and l_2. The magnetic field $B(t)$ is directed along the core in the direction of the vector $\mathbf{a}_S$. The width of the conducting coil is defined as w, and the length of the device is l_C.

As with the circular transformer emf device, the emf is generated in a conducting coil circuit due to a changing magnetic field linking the circuit. The magnetic field within the core is uniform, and varies sinusoidally, as shown for the circular design, given in equation (9).

Similar to the circular transformer emf device, the magnetic field within one or more cores changes due to the movement of magnetic devices with respect to the core. This is shown in Figure 8, in which a number of permanent magnets with alternating north–south pole faces moves, with velocity $\dot{X}$, relative to the ferromagnetic cores.

Using the same analysis as for the circular transformer emf device, the total, average power dissipation is given by

$$\bar{P}_{tran} = \frac{[NAB_0]^2\, 2\pi^2}{(R_{int} + R_{ext})} f_B^2 \quad \text{(W)}. \tag{33}$$

Figure 9 shows the "top view" of the rectangular transformer emf device.

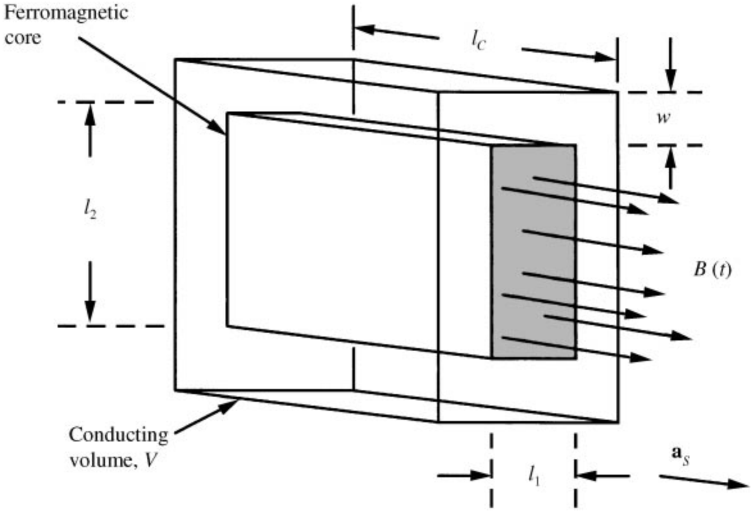

Figure 7. Rectangular "transformer emf" design.

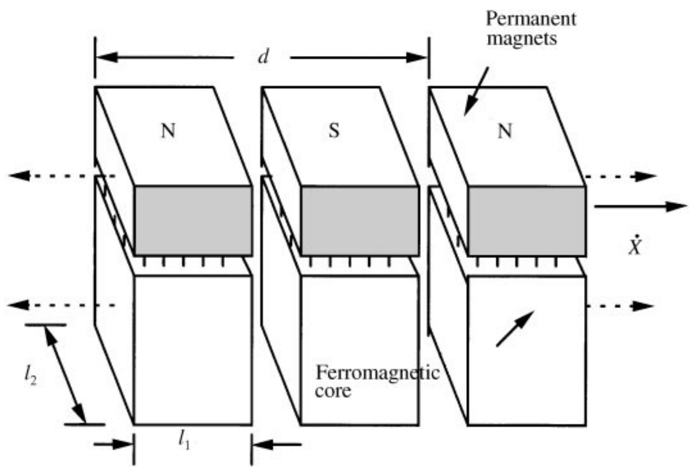

Figure 8. Core and permanent magnets in generalized rectangular "transformer emf" device.

Using the same analysis as for the circular "transformer emf" design, the maximum damping coefficient is given by

$$Ctran_{max} = \frac{2\pi^2}{d^2}\frac{[NAB_0]^2}{R_{int}} \quad (\mathrm{N\,s/m}). \tag{34}$$

The cross-sectional area of the core A, for the rectangular device is given by

$$A = l_1\, l_2 \quad (\mathrm{m}^2). \tag{35}$$

From Figure 9, the distance d, is minimized according to

$$d = 2(l_1 + 2w) \quad (\mathrm{m}). \tag{36}$$

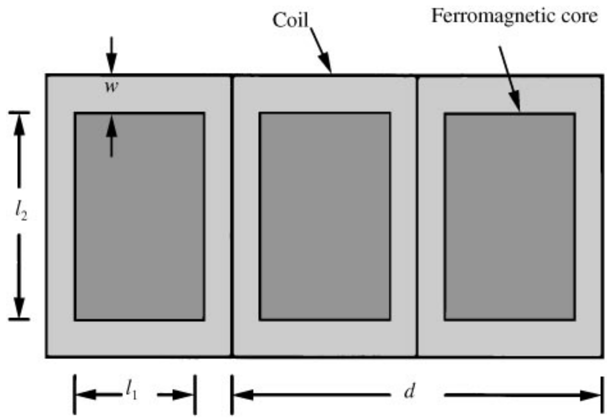

Figure 9. "Top view" transformer emf device.

Once again, the assumption is made that the conducting coil completely fills the volume defined as the coil region. Therefore, the conductor area a, the number of conducting turns N, the core length l_C, and the coil width w, can be related by

$$Na = l_C w \quad (\mathrm{m}^2). \tag{37}$$

The total conductor length l is given by

$$l = N2(l_1 + l_2 + 2w) \quad (\mathrm{m}), \tag{38}$$

and the total conductor resistance R_{int} is given by

$$R_{int} = \frac{l\sigma}{a} \quad (\Omega), \tag{39}$$

where σ is the conductivity of the conducting material. Substituting equations (35)–(39) into the damping coefficient function, equation (34) gives the damping coefficient with respect to the device dimensions,

$$Ctran = \frac{\pi^2 l_C w l_1^2 l_2^2}{2(2l_1 + 2l_2 + 4w)(l_1^2 + 4l_1 w + 4w^2)} \frac{B_0^2}{\sigma} \quad (\mathrm{N\,s/m}). \tag{40}$$

Given that the actual volume of conducting material for the transformer emf device is given by

$$V_{ACT} = 2l_C w(l_1 + l_2 + 2w) \quad (\mathrm{m}^3), \tag{41}$$

the efficiency of the transformer emf device, compared to the motional emf device, can be obtained by taking the ratio of their respective damping coefficients for the condition that the motional emf device has a device volume as given in equation (41). The relative efficiency is given by

$$H_C = \frac{Ctran}{Cmot} = \frac{\pi^2 l_1^2 l_2^2}{2(l_1 + l_2 + 2w)^2(2l_1 + 4w)^2}. \tag{42}$$

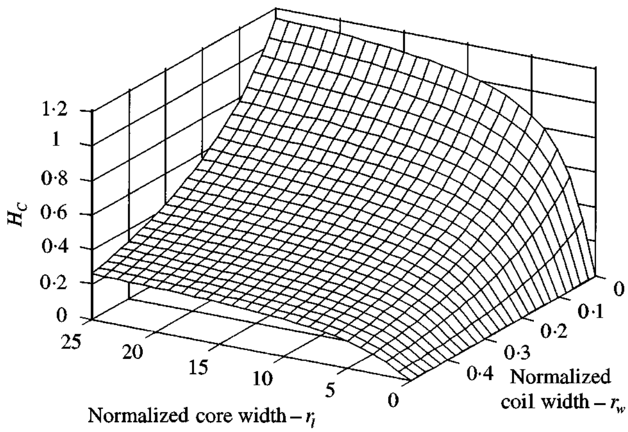

Figure 10. Rectangular "transformer emf" device efficiency.

In order to simplify the analysis, the parameter r_w is once again defined as the normalized conducting coil width, the ratio between the width of the conducting coil w, and the length of the core side, l_1:

$$r_w = w/l_1. \tag{43}$$

The parameter r_1 is defined as the normalized core width, the ratio between the length of the core sides l_1 and l_2,

$$r_1 = l_2/l_1. \tag{44}$$

Substituting the ratio given in equations (43) and (44) into equation (42) gives the relative efficiency between the two devices with respect to the normalized damper dimensions.

$$H_C = \frac{\pi^2 r_1^2}{2(1 + r_1 + 2r_w)^2(2 + 4r_w)^2}. \tag{45}$$

The relationship between the relative transformer emf device efficiency H_C, as a function of the normalized coil thickness r_w, and the normalized core width r_1, is shown in Figure 10.

Figure 10 reveals that by varying the dimensions of the transformer emf damper, it is possible to change the relative efficiency between the circular transformer emf device and the motional emf device. In fact, the efficiency of the transformer emf device can be greater than the motional emf design.

The maximum relative device efficiency occurs when r_w is zero, or the width of the conducting coils is zero, and the core side length l_2 is much larger than the core side length l_1. The maximum efficiency for the rectangular transformer emf device is $\pi^2/8$, or 123·37%. This is four times the maximum efficiency of the circular design. Similar to the circular transformer emf device, the relative efficiency reduces as the width of the conductors w increases.

The relative damper efficiency, defined in equation (45) also reveals that it is not possible to obtain a transformer emf damper efficiency greater than a motional emf device when the normalized coil width r_w is greater than 0·0554. This is important because for most realistic situations, the normalized coil width will be greater than this value. For a low relative coil width, the conducting coil only takes up a small proportion of the device volume, with a large volume needed for the ferromagnetic core. Therefore, for a given conductor volume, the total transformer emf device volume and mass need to be increased.

The reduction in the relative device efficiency, with respect to an increase in the relative coil width, occurs for the same reason as for the circular transformer emf device. This is because the change in magnetic field within the core, for a given device velocity, reduces as the relative coil width increases. For a similar reason, the relative device efficiency is increased when the core side length l_2 is much greater than the side length l_1. This is because for larger relative core width, there is a reduction of the distance between the magnetic pole faces d for a given conductor volume. This leads to an increase in the frequency of the magnetic field f_B for a given relative damper velocity $\dot{X}$. The increase in frequency leads to an increased power dissipation for a given damper relative velocity and, therefore, an increase in the damping coefficient.

5. DISCUSSION

In order to simplify the analysis in this paper, several simplifying assumptions have been made. The analysis of the motional emf electromagnetic devices assumed that the devices fully utilized the available magnetic field, or that the conductors were always within the maximum available magnetic field. This, however, is not always the case, as demonstrated by Karnopp [3], when analysing linear DC motors. Although Karnopp gave some suggestions as to a solution to this problem, the efficiency of many motional emf devices will be reduced due to this effect. Although many DC, rotating motional emf devices have the conductors totally within the magnetic field, a further problem arises due to the magnetic field, current and velocity vectors not being mutually perpendicular for all situations. This is another example of the reduction in the "ideal" efficiency of motional emf devices, as evaluated in Section 2. This will increase the relative damping efficiency of the transformer emf devices.

There were also several simplifying assumptions made for the transformer emf devices. The assumption of a uniform magnetic field within the core of these devices, and the magnitude of the sinusoidally varying magnetic field were the two important assumptions. Both of these assumptions need to be further analyzed, both theoretically and experimentally, in order to verify this assumption for particular circumstances. Further analysis is needed to verify the overall relative damping results of the transformer emf device. This is to verify the results over a range of conditions, and to compare the results with realistic, experimental models.

6. CONCLUSION

A comparison between electromagnetic dampers based on motional emf and transformer emf principles has been presented. It was found that transformer emf devices with circular cores had a maximum device efficiency of 30·84% compared with motional emf devices. It was demonstrated that by modifying the device dimensions, and flattening the core, it is possible to increase the relative efficiency of the transformer emf device up to a maximum of 123·37%. The theoretical analysis revealed, however, that in almost all realistic situations, the motional emf designs will have a greater damping efficiency than the transformer emf designs. This is because, to increase the transformer emf efficiency, the device dimensions are constrained, which leads to an increase in the overall device volume and mass. Therefore for applications such as regenerative and semi-active damping in vehicle suspension systems, motional emf devices are more suitable, due to their improved damping efficiency.

ACKNOWLEDGMENTS

The authors would like to express their appreciation to Tim Petherbridge and Canticle of the Sun Company Ltd. for the support for research conducted in association with this publication.

REFERENCES

1. Y. OKADA, J. YONEMURA and M. SHIBATA 1997 *ASME Design Engineering Technical Conference*. Regenerative damper with an optimally tuned resonant circuit.
2. Y. OKADA, J. YONEMURA and M. SHIBATA 1998 *The 4th International Symposium on Motion and Vibration Control*, 85–90. Regenerative control of moving mass type vibration damper.
3. D. KARNOPP 1989 *Vehicle System Dynamics* **18**, 187–200. Permanent magnet linear motors used as variable mechanical dampers for vehicle suspensions.
4. Y. SUDA and T. SHIIBA 1996 *Vehicle System Dynamics Supplement* **25**, 641–654. A new hybrid suspension system with active control and energy regeneration.
5. T. OKADA and H. HARADA 1996 *Proceedings of the 35th Conference on Decision and Control*, 4715–4720. Regenerative control of active vibration damper and suspension systems.
6. Y. SUDA, S. NAKADAI and K. NAKANO 1998 *The 4th International Symposium on Motion and Vibration Control*, 91–96. Study on the self-powered active vibration control.
7. Y. SUDA, S. NAKADAI, K. NAKANO, H. TSUNASHIMA and T. WASHIZU 1998 *Proceedings of the 4th International Symposium on Advanced Vehicle Control*, 425–430. Self-powered active control applied to a cab suspension.
8. Y. OKADA, K. MATSUDA and H. HASHITANI 1995 *Journal of Vibration and Acoustics* **117**, 411–415. Self-sensing active vibration control using the moving-coil-type actuator.
9. D. CHENG 1989 *Field and Wave Electromagnetics*. New York: Addison-Wesley.